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365. Proposed by C. N. SCHMALL, New York City.

Show that the area inclosed by each of the following three curves is equal to the circle of radius a; viz., πa^2.

(1) $a^2x^2 = y^3(2a - y)$, (2) $a^2 - x^2 = (y - mx^2)^2$, (3) $(xy + c + bx^2)^2 = x^2(a^2 - x^2)$.

I. Solution by A. M. Harding, University of Arkansas.

If we change these equations to parametric forms we obtain

(1) $x = 4a \cos^3 t \sin t, \quad y = 2a \cos^2 t,$

(2) $x = a \sin t, \quad y = ma^2 \sin^2 t + a \cos t,$

(3) $x = a \sin t, \quad y = \dfrac{a^2 \cos t - a^2 b \sin t - c \csc t}{a}.$

Hence,

(1) $\text{Area} = \int y\, dx = \int_0^{\pi} 8a^2(4 \cos^6 t - 3 \cos^4 t)dt = \pi a^2;$

(2) $\text{Area} = \int y\, dx = \int_0^{2\pi} (ma^2 \sin^2 t + a \cos t)a \cos t\, dt = \pi a^2;$ and

(3) $\text{Area} = \int y\, dx = \int_0^{2\pi} (a^2 \cos t - a^2 b \sin t - c \csc t) \cos t\, dt = \pi a^2.$

II. Solution by H. C. Feemster, York College, Nebraska.

$$A_1 = \int_0^{2a} \int_{-(y/a)\sqrt{2ay - y^2}}^{(y/a)\sqrt{2ay - y^2}} dydx = \left[-\frac{3a^2 + ax - 2x^2}{3} \sqrt{2ax - x^2} + a^2 \operatorname{vers}^{-1} \frac{x}{a} \right]_0^{2a} = \pi a^2;$$

$$A_2 = \int_{-a}^{a} \int_{mx - \sqrt{a^2 - x^2}}^{mx + \sqrt{a^2 - x^2}} dxdy = \left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a} \right]_{-a}^{a} = \pi a^2;$$

and

$$A_3 = \int_{-a}^{a} \int_{-\sqrt{a^2 - x^2} - (c/x) - bx}^{\sqrt{a^2 - x^2} - (c/x) - bx} dxdy = \left[x\sqrt{a^2 - x^2} + a^2 \sin^{-1} \frac{x}{a} \right]_{-a}^{a} = \pi a^2.$$

NUMBER THEORY.

218. Proposed by ELIJAH SWIFT, University of Vermont.

If p is prime > 3 show that

$$\sum_{a=1}^{a=p-1} \frac{1}{a^2} \equiv 0 \pmod{p}. \tag{1}$$

I. Solution by Tracy A. Pierce, Berkeley, Cal.

In (1), we may replace 1 by a^{p-1}, since $a^{p-1} \equiv 1 \pmod{p}$. We then have

$$\sum_{a=1}^{a=p-1} a^{p-3} \equiv 0 \pmod{p}.$$